GCSE Maths – Number

Factors, Multiples and Primes

Notes

WORKSHEET

Factors

A factor is a number which **divides** into another number **exactly**, without leaving a remainder. The factors of a number are always less than or equal to the number itself. For example, the factors of **8** are **1, 2, 4, 8**.

Finding Factors

The easiest way to find factors is to start with 1 and work your way up.
On the right is a table of **factors of 16**.

1	16
2	8
4	4

Factors must come in **pairs** as multiplying the factor in the left hand side of the table with the factor on the right hand side of the table produces the number being factored.

Common Factors

You can find the common factors of two or more numbers. To do this, **list all the factors** of each number and then compare them to see which factors they have in common. The **highest common factor (HCF)** is the common factor of highest value.

Example: Find the highest common factor of 40 and 24

Factors of 40: 1, 2, 4, 5, 8, 10, 20

Factors of 24: 1, 2, 3, 4, 6, 8, 12

The numbers 24 and 40 have common factors 1, 2, 4, 8.

Therefore, their highest common factor is 8.

Multiples

Multiples are numbers which have the **original number as a factor**. Multiples of a number are easily found by multiplying the number by any other integer. For example, the multiples of **12** are **12, 24, 36, 48,** …

Common multiples

A common multiple is a number that is a **shared multiple** of two or more numbers. The **lowest common multiple (LCM)** is the common multiple of the lowest value.

Example: Find the lowest common multiple of 9 and 12

*Multiples of 9: 9, 18, 27, **36**, 45, 54,...*

*Multiples of 12: 12, 24, **36**, 48, …*

The numbers 9 and 12 have 36 as their lowest common multiple.

Primes

A prime number is a number which is **divisible by only 1** and **itself**. This means prime numbers have **two distinct factors**. The number 1 is not a prime number because it only has one distinct factor.

Which numbers are prime?

- **1** is not a prime number because it only has one distinct factor.
- **2** and **3** are both prime because they each are only divisible by 1 and themselves.
- **4** is not prime because it is divisible by 2 so it has three distinct factors.
- None of the **even numbers** greater than 2 are prime because they are all divisible by 2. This means that 2 is the only even prime number.
- **9** is not prime because it is divisible by 3.

The first ten primes are: **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...**

Prime Factorisation

All numbers have a **unique prime factorisation**. This means that every number can be written as a product of prime numbers.

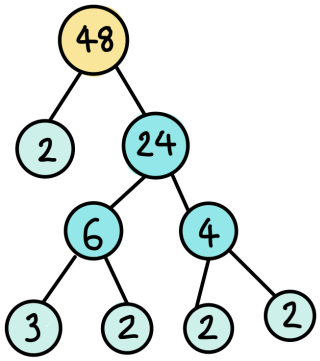
For example,

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$

The prime factorisation of a number can be found using a **factor tree**.

The **factor tree** is produced in stages:

- First the number is divided into two factors.
- These factors are then divided into two further factors and so on.
- The branches of the tree stop when a prime number is reached as at this point there are no further factors.

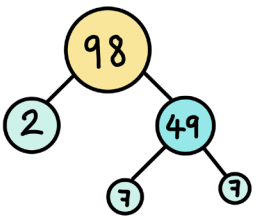


The **prime factorisation** of a **prime number** is itself.

Example: Find the prime factorisation of 98

*Use a **factor tree** to find the prime factorisation: First, divide 98 into the factors 49 and 2. Since 2 is prime, the branch with 2 stops. Since 49 has two factors of 7, it reduces into two further branches of 7, which is prime. So the prime factorisation is*

$$98 = 2 \times 7 \times 7 = 2 \times 7^2.$$



Primes, Factors and Multiples - Practice Questions

1. Find the common factors of 27 and 45.

2. Find the lowest common factor of 88, 50, 65.

3. Find the highest common factor of x and y where:

$$x = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$
$$y = 2 \times 3 \times 3 \times 5$$

4. Find the lowest common multiple of 3, 4 and 9.

5. Find two numbers with the lowest common multiple of 36.

6. A blue light flashes every 8 seconds, a red light flashes every 12 seconds and a green light flashes every 14 seconds.
   a) After how much time does all the three lights flash together?
   b) When will all the lights flash together for the third time?

7.
   a) Find the prime factorization of 45 and 60.
   b) Using answer to part a), find the highest common factor of 45 and 60.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.